Monte Hall with a Twist

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Fun time. We all know the original MH problem – three closed doors behind one of which is a car (C) and goats behind the other two. You pick one door, MH knows the C-door and opens one of the other doors with a goat. You now get to stay with the original door or switch to the remaining unopened door. A lot of clever people made fools of themselves saying switching didn't matter, since the probability of picking the C-door from the remaining two closed doors was 50-50. We now know that switching ups your probability of winning the car from 1/3 to 2/3.

So I was thinking of an interesting variation of the MH problem. You now have $N > 3$ doors with C behind one of them. You pick one, and then MH opens $M < N$-2 doors with goats staring at you. What's the probability of winning C by switching to one of the doors unopened by MH, and what is the probability of winning C by staying with your original pick?

Notationally $P(C|N,M)$ is the probability of winning C when switching given N and M. And $P(C|N)$ is the probability of winning, given N, when you stick with your original pick – i.e. you don't switch. Hint: $P(C|N)$ is the same probability as C *not* being behind one of the remaining doors that MH can open. Formally we write $P(C|N) = P(\neg C|N,M)$. The minus sign with the hangy-down part is the symbol for the logical NOT. So, the probability of C not being behind one of the N-1 doors that you did not select is the same as its being behind the door that you did not select. Capice?

Final extra credit question – if C is valued at \$100K, what is the maximum amount you should be willing to pay to play the game? So here is the solution.

$$P(C \mid N) = P(\neg C \mid N, M) = \text{\{Prob no C behind a remaining unoperned doors\}} = \left(\frac{1}{N}\right)$$

$$P(C \mid N, M) = \text{\{Prob C behind a remaining unoperned doors\}}\,\text{\{Prob C behind a remaining unoperned door\}}$$

$$= \left(\frac{N-1}{N}\right)\left(\frac{1}{N-1-M}\right) = \frac{1}{N}\left(\frac{N-1}{N-1-M}\right)$$

Confirmed that for $N \geq 3, M > 0$ we must have $P(C \mid N, M) + P(\neg C \mid N, M) = 1$.

And note that for $M > 0$ we should always switch since $P(C \mid N, M) > P(\neg C \mid N, M)$.

Finally, if the value of C is $\$VC$ = \$100K in a game where N = 100 and M = 80, and you switch your door pick, then the expected value of the game is the maximum you should pay to play. The expected game value $\$VG = P(C|N,M)*\$VC$ = 0.0521*\$100K = \$5,210. Most people would not go that high. What amount would you be willing to pay for play?